INTERNET DRAFT
draft-ohta-qec-inapplicable-00.txt
Intended status: Informational
Expires: May 3, 2021

M. Ohta
Tokyo Institute of Technology

October 30, 2020

Quantum Error Correction Inapplicable to Really Entangled States

Status of this Memo

Copyright Notice

Abstract

Though quantum error correction assumes localized error model of Shor that errors on a qubit are caused by interaction with its local environment, enabling essentially classical error correction for unentangled states, the model is applied to entangled states improperly without involving local environment states in the entanglement.

That is, when an entangled state (Q) is represented as superposition of unentangled terms (Qi) as Q=Q1+Q2+...+Qn, local environment states around qubits are, in general, different term by term. Q will be, with term-specific error operators (Ei), E1*Q1+E2*Q2+...+En*Qn, not, with a common error operator (E) assumed by Shor, E*(Q1+Q2+...+Qn).

A complication is that Shor's error model is a little quantum, allowing for two different local environment states around a qubit. As such, quantum error correction is applicable to some trivially entangled states including states used by Shor code but not to really entangled states.

1. Introduction

An assumption of noise model for quantum error correction by Shor [1] is "The critical assumption here is that decoherence only affects one qubit of our superposition, while the other qubits remain unchanged. It is not clear how reasonable this assumption is physically, but it corresponds to the assumption in classical information theory of the independence of noise.", which means a qubit suffers from error as a result of interaction with local environment around the qubit but no interaction occurs with other qubits or local environment of other qubits. Though some extension to consider certain interaction between a qubit and other qubits or environment of other qubits is possible, some locality is still assumed.

The error model is directly applicable to unentangled, that is, essentially classical, states, resulting in localized errors, corrections of which are essentially classical error correction.

However, it is unreasonable to expect such localized errors for entangled states, because the states themselves do not have locality. Actually, with a 2 qubit entangled state: |00>+|11>, if the first qubit coherently interacts with its environment to be |0>, the entire state becomes |00>, which means the second qubit is also affected, Though the case is trivial enough to be explained by Shor's error model as superposition of identity (no error) and sign flip (|0> and |1> become |0> and -|1>, correspondingly) error: |00>=((|00>+|11>)+(|00>-|11>))/2, such an explanation dose not deny lack of locality of errors on entangled states.

As Shor overlooked the fact that when qubit states are entangled, their environment states are, in general, also entangled, errors on really entangled states are highly non-local to which quantum error correction is not applicable.

That is, when an entangled state (Q) is represented as superposition of (minimum number of) unentangled terms (Qi) as Q=Q1+Q2+...+Qn, local environment states around a qubit are, in general, involved in the entanglement and different term by term, resulting in different error operators (Ei). As a result, Q will be disturbed by noise to be E1*Q1+E2*Q2+...+En*Qn, whereas, Shor thought a common error operator (E) is applicable to all the terms as E*(Q1+Q2+...+Qn).

It is obvious that, with some clever encoding using fixed number of extra qubits, effect of E may be compensated, which was quantum error correction, but the extra qubits are not enough to compensate all the Ei's (with quantum algorithms, 'n' will often be exponentially large w.r.t. problem size).

A complication is that Shor's error model is a little quantum, allowing for, seemingly despite his intention, two different local environment states around a qubit, which is explained in the next section.

2. Why Shor's Error Model is a little Quantum?

In [1], Shor explicitly described environment state of a qubit before interaction with the qubit |e0> (same state for |0> and |1>, which should be the intention of Shor) and described interaction (decoherence) process as:

|e0>|0> -> |a0>|0>+|a1>|1>

|e0>|1> -> |a2>|0>+|a3>|1>

where |a0>, |a1>, |a2> and |a3> are environment states after the interaction. |a0>, |a1>, |a2> and |a3> are "not generally orthogonal or normalized" [1] and can be fully independent each other. Ignoring error terms,

|e0>|0> -> |a0>|0>

|e0>|1> -> |a3>|1>

So, if qubit state is |0>, its environment state is |a0>, but, if qubit state is |1>, its environment state us |a3>, different from |a0>.

It should also be noted that, as |a0>, |a1>, |a2> and |a3> are fully independent each other, the process may have two different initial environment states as:

|e0>|0> -> |a0>|0>+|a1>|1>

|e1>|1> -> |a2>|0>+|a3>|1>

So, Shor's error model is slightly quantum allowing for different environment states depending on qubit values.

As such, errors on trivially entangled states (e.g., superposition of just two unentangled states) such as |00>+|11> and

(|000>+|111>)(|000>+|111>)(|000>+|111>) should be correctable. As the latter example is Shor code for |0>, experimental confirmation of Shor's quantum error correction should success, as long as the input qubit to an error correction circuit is unentangled with other qubits outside of the circuit, which is not the case when quantum algorithms are run on quantum computers relying on aggressive entanglement between qubits.

It should be noted that, though it does not affect the points of this memo, Shor's representation of qubit and its environment states using tensor product is inappropriate, because, for the interaction, relative phase between them matters (e.g., resulting states of homodyne detection relies on the relative phase), which can be represented by not tensor but Cartesian product. Though |e0>|0> and -|e0>|0> represent a same state, (|e0>, |0>) and (|e0>, -|0>) are different states.

It should also be noted that Shor's error model is a little quantum not because sign flip error is quantum specific and classically impossible. It is merely that sign flip error does not occur on modern computers where phase is not used to encode information. In an optical packet router using FDLs (Fiber Delay Lines) as optical buffers (memory), like ancient computers with Mercury delay lines as memory, where QAM (Quadrature Amplitude Modulated) PDM (Polarization Division Multiplexed) signal is sent over the FDLs [2], sign flip errors occur as relative phase errors between polarization modes.

3. Conclusions

It is shown that not-really-quantum error correction works only for errors with mostly classical locality and is not applicable to non-local errors on really entangled states.

As qubit states within quantum computers running quantum algorithms are really entangled, quantum error correction for them is impossible, which makes construction of quantum computers with practical size practically impossible.

Entangled states, in general, are a lot noisier than Shor thought, which should be the reason why the states are so fragile easily collapsing to be less noisy less entangled or unentangled states.

4. Security Considerations

That construction of quantum computers with practical size is practically impossible means quantum computers do not make public key cryptography unsafe, though there may still be some classical algorithm to make it unsafe.

5. IANA Considerations

This memo has no actions for IANA.

Informative References

[1] P. W. Shor, "Scheme for reducing decoherence in quantum computer memory", Phys. Rev. A, Oct. 1995, http://www.cs.miami.edu/~burt/learning/Csc670.052/pR2493_1.pdf.

[2] M. Ohta, "Optical switching of many wavelength packets: A conservative approach for an energy efficient exascale interconnection network", 2016 IEEE 17th International Conference on High Performance Switching and Routing (HPSR), https://ieeexplore.ieee.org/document/7525641, August 2016.

Author's Address

Masataka Ohta
Tokyo Institute of Technology
2-12-1-W8-54, O-okayama, Meguro-ku
Tokyo 152-8552
JAPAN

Phone: +81-3-5734-3299
Fax: +81-3-5734-3299
EMail: mohta@necom830.hpcl.titech.ac.jp