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Schnorr NIZK Proof: Non-interactive Zero Knowledge Proof for Discrete Logarithm draft-hao-schnorr-01

Abstract

This document describes the Schnorr NIZK proof, a non-interactive variant of the three-pass Schnorr identification scheme. The Schnorr NIZK proof allows one to prove the knowledge of a discrete logarithm without leaking any information about its value. It can serve as a useful building block for many cryptographic protocols to ensure the participants follow the protocol specification honestly.

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1. Introduction

A well-known principle for designing robust public key protocols states as follows: "Do not assume that a message you receive has a particular form (such as g^r for known r) unless you can check this" [<u>AN95</u>]. This is the sixth of the eight principles defined by Ross Anderson and Roger Needham at Crypto'95. Hence, it is also known as the "sixth principle". In the past thirty years, many public key protocols failed to prevent attacks, which can be explained by the violation of this principle [<u>Hao10</u>].

While there may be several ways to satisfy the sixth principle, this document describes one technique that allows one to prove the knowledge of a discrete logarithm (i.e., r for g^r) without revealing its value. This technique is called the Schnorr NIZK proof, which is a non-interactive variant of the three-pass Schnorr identification scheme [Stinson06]. The original Schnorr identification scheme is made non-interactive through a Fiat-Shamir transformation [FS86], assuming that there exists a secure cryptographic hash function.

<u>1.1</u>. Requirements language

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in <u>RFC 2119</u> [<u>RFC2119</u>].

<u>1.2</u>. Notations

The following notations are used in this document:

- o Alice: the assumed identity of the prover in the protocol
- o Bob: the assumed identity of the verifier in the protocol
- o p: a large prime
- o q: a large prime divisor of p-1
- o Zp*: a multiplicative group of integers modulo p
- o Gq: a subgroup of Zp* with prime order q
- o g: a generator of Gq
- o g^r: the base g raised to the power of r
- o a mod b: a modulo b
- o a * b: a multiplied by b
- o a || b: concatenation of a and b
- o t: the bit length of the challenge chosen by Bob
- o H: a secure cryptographic hash function

2. Group parameters

The Schnorr NIZK proof uses exactly the same group setting as DSA (or ECDSA). For simplicity, this document will only describe the Schnorr NIZK proof in the DSA-like group setting. The technique works basically the same in the ECDSA-like group, except that the underlying multiplicative cyclic group is replaced by an additive cyclic group defined over some elliptic curve.

The DSA group setting consists of three parameters: (p, q, g). NIST has published a set of values for (p, q, g) for different security levels; they can be found in <u>Appendix A</u>. The same values can be used for the Schnorr NIZK proof.

3. Schnorr Identification Scheme

The Schnorr identification scheme is a zero-knowledge proof primitive that allows one to prove the knowledge of the discrete logarithm

without leaking any information about its value. It has been proven secure assuming that the verifier is honest and that the discrete logarithm problem is intractable [Stinson06]. This scheme runs interactively between Alice (prover) and Bob (verifier). In the setup of the identification scheme, Alice has published her public key X = g^x mod p where x is the private key chosen uniformly at random from [0, q-1]. The value X must be an element in the subgroup Gq, which anyone can verify. This is to ensure that the discrete logarithm of X with respect to the base g actually exists.

The protocol works in thee passes:

- 1. Alice chooses a number v uniformly at random from [0, q-1] and computes $V = q^v mod p$. She sends V to Bob.
- Bob chooses a challenge h uniformly at random from [0, 2^t 1], where t is the bit length of the challenge (say t = 80). Bob sends h to Alice.
- 3. Alice computes $b = v x * h \mod q$ and sends it to Bob.

At the end of the protocol, Bob checks if the following equality holds: $V = g^b * X^h \mod p$. The verification succeeds only if the equality holds. The process is summarized in the following diagram.

Information Flows in Schnorr Identification Scheme

| Alice | Bob |
|-------------------------------|--|
| | |
| choose random v from [0, | q-1] |
| compute $V = g^v \mod p$ | V -> |
| compute $b = v - x^*h \mod q$ | <- h choose random h from [0, 2 ^t -1] |
| | b -> check if V = g^b * X^h mod p? |

<u>4</u>. Schnorr NIZK proof

The Schnorr NIZK proof is obtained from the interactive Schnorr identification scheme through a Fiat-Shamir transformation [FS86]. This transformation involves using a secure cryptographic hash function to issue the challenge instead. More specifically, the challenge is redefined as $h = H(g || g^v || g^x || UserID ||$ OtherInfo), where UserID is a unique identifier for the prover and OtherInfo is optional data. The OtherInfo is included here for generality, as some security protocols built on top of the Schnorr

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Schnorr NIZK Proof

NIZK proof may wish to include more contextual information such as the protocol name, timestamp and so on. The exact items (if any) in OtherInfo will be left to specific protocols to define. However, the format of OtherInfo in any specific protocol must be fixed and explicitly defined in the protocol specification.

Within the hash function, there must be a clear boundary between the concatenated items. Usually, the boundary is implicitly defined once the length of each item is publicly known. However, in the general case, it is safer to define the boundary explicitly. It is recommended that one should always prepend each item with a 4-byte integer that represents the byte length of the item. The OtherInfo may contain multiple sub-items. In that case, the same rule shall apply to ensure a clear boundary between adjacent sub-items.

5. Computation cost of Schnorr NIZK proof

In summary, to prove the knowledge of the exponent for $X = g^x$, Alice generates a Schnorr NIZK proof that contains: {UserID, OtherInfo, V = $g^v \mod p$, $r = v - x^{*} \mod q$ }, where $h = H(g \parallel g^v \parallel g^x \parallel g^x \parallel UserID \parallel OtherInfo)$.

To generate a Schnorr NIZK proof, the cost is roughly one modular exponentiation: that is to compute g^v mod p. In practice, this exponentiation can be pre-computed in the off-line manner to optimize efficiency. The cost of the remaining operations (random number generation, modular multiplication and hashing) is negligible as compared with the modular exponentiation.

To verify the Schnorr NIZK proof, the following computations shall be performed.

1. To verify X is within [1, p-1] and X^q = 1 mod p

2. To verify $V = g^r * X^h \mod p$

Hence, the cost of verifying a Schnorr NIZK proof is approximately two exponentiations: one for computing X^q mod p and the other for computing g^r * X^h mod p. (It takes roughly one exponentiation to compute the latter using a simultaneous exponentiation technique as described in [MOV96].)

It is worth noting that some applications may specifically exclude the identity element as a valid public key. In that case, one shall check X is within [2, p-1] instead of [1, p-1]. Also note that in the DSA-like group setting, it requires a full modular exponentiation to validate a public key, but in the ECDSA-like setting, the public key validation incurs almost negligible cost due to the co-factor being very small (see [MOV96]).

<u>6</u>. Applications of Schnorr NIZK proof

Some key exchange protocols, such as J-PAKE [HR08] and YAK [Hao10], rely on the Schnorr NIZK proof to ensure participants in the protocol follow the specification honestly. Hence, the technique described in this document can be directly applied to those protocols.

The inclusion of OtherInfo also makes the Schnorr NIZK proof generally useful and sufficiently flexible to cater for a wide range of applications. For example, the described technique may be used to allow a user to demonstrate the Proof-Of-Possession (PoP) of a longterm private key to a Certificate Authority (CA) during the public registration phrase. Accordingly, the OtherInfo may include extra information such as the CA name, the expiry date, the applicant's email contact and so on. In this case, the Schnorr NIZK proof is essentially no different from a self-signed Certificate Signing Request generated by using DSA (or ECDSA). The details are however beyond the scope of this document.

7. Security Considerations

The Schnorr identification protocol has been proven to satisfy the following properties, assuming that the verifier is honest and the discrete logarithm problem is intractable (see [Stinson06]).

- 1. Completeness -- a prover who knows the discrete logarithm is always able to pass the verification challenge.
- Soundness -- an adversary who does not know the discrete logarithm has only a negligible probability (i.e., 2^{-t}) to pass the verification challenge.
- Honest verifier zero-knowledge -- a prover leaks no more than one bit information to the honest verifier: whether the prover knows the discrete logarithm.

The Fiat-Shamir transformation is a standard technique to transform a three-pass interactive Zero Knowledge Proof protocol (in which the verifier only chooses a random challenge) to a non-interactive one, assuming that there exists a secure cryptographic hash function.

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Since the hash function is publicly defined, the prover is able to compute the challenge by herself, hence making the protocol non-interactive. The assumption of an honest verifier naturally holds because the verifier can be anyone.

A non-interactive Zero Knowledge Proof is often called a signature scheme. However, it should be noted that the Schnorr NIZK proof described in this document is different from the original Schnorr signature scheme (see [Stinson06]) in that it is specifically designed as a proof of knowledge of the discrete logarithm rather than a general-purpose digital signing algorithm.

When a security protocol relies on the Schnorr NIZK proof for proving the knowledge of a discrete logarithm in a non-interactive way, the threat of replay attacks shall be considered. For example, the Schnorr NIZK proof might be replayed back to the prover herself (to introduce some undesirable correlation between items in a cryptographic protocol). This particular attack is prevented by the inclusion of the unique UserID into the hash. The verifier shall check the prover's UserID is a valid identity and is different from her own. Depending the context of specific protocols, other forms of replay attacks should be considered, and appropriate contextual information included into OtherInfo whenever necessary. The UserID and items (if any) in OtherInfo shall be left to specific protocols to define.

8. IANA Considerations

This document has no actions for IANA.

9. Acknowledgements

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10. References

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- [MOV96] Menezes, A., Oorschot, P., and S. Vanstone, "Handbook of Applied Cryptography", 1996.

[Stinson06]

Stinson, D., "Cryptography: Theory and Practice (3rd Edition)", CRC, 2006.

<u>10.2</u>. Informative References

- [HR08] Hao, F. and P. Ryan, "Password Authenticated Key Exchange by Juggling", the 16th Workshop on Security Protocols, May 2008.
- [Hao10] Hao, F., "On Robust Key Agreement Based on Public Key Authentication", the 14th International Conference on Financial Cryptography and Data Security, February 2010.

<u>Appendix A</u>. Group parameters

The Schnorr NIZK proof operates in exactly the same cyclic groups as the Digital Signature Algorithm (DSA). The following 1024-, 2048-, 3072-bit groups are public domain parameters published by NIST for the DSA. They are taken from the following address:

http://csrc.nist.gov/groups/ST/toolkit/documents/Examples/ DSA2_All.pdf

1. 1024-bit p and 160-bit q

p =

E0A67598CD1B763BC98C8ABB333E5DDA0CD3AA0E5E1FB5BA8A7B4EABC10BA338FAE06DD4B90FDA70D7CF0CB0C638BE3341BEC0AF8A7330A3307DED2299A0EE606DF035177A239C34A912C202AA5F83B9C4A7CF0235B5316BFC6EFB9A248411258B30B839AF172440F32563056CB67A861158DDD90E6A894C72A5BBEF9E286C6B555

q = E950511E AB424B9A 19A2AEB4 E159B784 4C589C4F

g = D29D5121 B0423C27 69AB2184 3E5A3240 FF19CACC 792264E3 BB6BE4F7 8EDD1B15 C4DFF7F1 D905431F 0AB16790 E1F773B5 CE01C804 E509066A 9919F519 5F4ABC58 189FD9FF 987389CB 5BEDF21B 4DAB4F8B 76A055FF

E2770988 FE2EC2DE 11AD9221 9F0B3518 69AC24DA 3D7BA870 11A701CE 8EE7BFE4 9486ED45 27B7186C A4610A75

```
2. 2048-bit p and 224-bit q
```

```
p =
```

C196BA05AC29E1F9C3C72D56DFFC6154A033F1477AC88EC37F09BE6C5BB95F51C296DD20D1A28A067CCC4D4316A4BD1DCA55ED1066D438C35AEBAABF57E7DAE428782A95ECA1C143DB701FD48533A3C18F0FE23557EA7AE619ECACC7E0B51652A8776D02A425567DED36EABD90CA33A1E8D988F0BBB92D02D1D20290113BB562CE1FC856EEB7CDD92D33EEA6F410859B179E7E789A8F75F645FAE2E136D252BFFAFF89528945C1ABE705A38DBC2D364AADE99BE0D0AAD82E5320121496DC65B3930E38047294FF877831A16D5228418DE8AB275D7D75651CEFED65F78AFC3EA7FE4D79B35F62A0402A1117599ADAC7B269A59F353CF450E6982D3B1702D9CA83

q =

90EAF4D1 AF0708B1 B612FF35 E0A2997E B9E9D263 C9CE6595 28945C0D

```
g =
```

A59A749A11242C58C894E9E5A91804E8FA0AC64B56288F8D47D51B1EDC4D65444FECA0111D78F35FC9FDD4CB1F1B79A3BA9CBEE83A3F811012503C8117F98E5048B089E387AF6949BF8784EBD9EF45876F2E6A5A495BE64B6E770409494B7FEE1DBB1E4B2BC2A53D4F893D418B7159592E4FFFDF6969E91D770DAEBD0B5CB14C00AD68EC7DC1E5745EA55C706C4A1C5C88964E34D09DEB753AD418C1AD0F4FDFD049A955E5D78491C0B7A2F1575A008CCD727AB376DB6E695515B05BD412F5B8C2F4C77EE10DA48ABD53F5DD498927EE7B692BBBCDA2FB23A516C5B4533D73980B2A3B60E384ED200AE21B40D273651AD6060C13D97FD69AA13C5611A51B9085

3. 2048-bit p, 256-bit q

p =

F56C2A7D366E3EBDEAA1891FD2A0D099436438A673FED4D75F594959CFFEBCA7BE0FC72E4FE67D91D801CBA0693AC4ED9E411B41D19E2FD1699C4390AD27D94C69C0B143F1DC88932CFE2310C886412047BD9B1C7A67F8A25909132627F51A0C866877E672E555342BDF9355347DBD43847156B2C20BAD9D2B071BC2FDCF9757F75C168C5D9FC43131BE162A0756D1BDEC2CA0EB0E3B018A8B38D3EF2487782AEB9FBF99D8B30499C55E4F61E5C7DCEE2A2BB55BD7F75FCDF00E48F2E8356BDB59D86114028F67B8E07B127744778AFF1CF1399A4D679D92FDE7D941C5C85C5D7BFF91BA69F9489D531D1EBFA727CFDA651390F8021719FA9F7216CEB177BD75SSSSSSSSS

q =

C24ED361 870B61E0 D367F008 F99F8A1F 75525889 C89DB1B6 73C45AF5 867CB467

g =

 8DC6CC81
 4CAE4A1C
 05A3E186
 A6FE27EA
 BA8CDB13
 3FDCE14A
 963A92E8

 09790CBA
 096EAA26
 140550C1
 29FA2B98
 C16E8423
 6AA33BF9
 19CD6F58

 7E048C52
 666576DB
 6E925C6C
 BE9B9EC5
 C16020F9
 A44C9F1C
 8F7A8E61

 1C1F6EC2
 513EA6AA
 0B8D0F72
 FED73CA3
 7DF240DB
 57BB8274
 31D61869

 7B9E771B
 0B301D5D
 F0595542
 5061A30D
 C6D33BB6
 D2A32BD0
 A75A0A71

 D2184F50
 6372ABF8
 4A56AEEE
 A8EB693B
 F29A6403
 45FA1298
 A16E8542

 1B2208D0
 0068A5A4
 2915F82C
 F0B858C8
 FA39D43D
 704B6927
 E0B2F916

 304E86FB
 6A1B487F
 07D8139E
 428BB096
 C6D67A76
 EC0B8D4E
 F274B8A2

 CF556D27
 9AD267CC
 EF5AF477
 AFED029F
 485B5597
 739F5D02
 40F67C2D

 948A6279
 94
 94
 94
 94567C2
 940F67C2D

4. 3072-bit p, 256-bit q

```
p =
```

90066455B5CFC38F9CAA4A48B4281F292C260FEEF01FD61037E56258A7795A1C7AD46076982CE6BB956936C6AB4DCFE05E6784586940CA544B9B2140E1EB523F009D20A7E7880E4E5BFA690F1B9004A27811CD9904AF70420EEFD6EA11EF7DA129F58835FF56B89FAA637BC9AC2EFAAB903402229F491D8D3485261CD068699B6BA58A1DDBBEF6DB51E8FE34E8A78E542D7BA351C21EA8D8F1D29F5D5D15939487E27F4416B0CA632C59EFD1B1EB66511A5A0FBF615B766C5862D0BD8A3FE7A0E0DA0FB2FE1FCB19E8F9996A8EA0FCCDE538175238FC8B0EE6F29AF7F642773EB8CD5402415A01451A840476B2FCEB0E388D30D4B376C37FE401C2A2C2F941DAD179C540C1C8CE030D460C4D983BE9AB0B20F69144C1AE13F9383EA1C08504FB0BF321503EFE43488310DD8DC77EC5B8349B8BFE97C2C560EA878DE87C11E3D597F1FEA742D73EEC7F37BE43949EF1A0D15C3F3E3FC0A8335617055AC91328EC22B50FC15B941D3D1624CD88BC25F3E941FDDC6200689581BFEC416B4B2CB73AA

q =

CFA0478A 54717B08 CE64805B 76E5B142 49A77A48 38469DF7 F7DC987E FCCFB11D

g =

5E5CBA992E0A680D885EB903AEA78E4A45A469103D448EDE3B7ACCC54D521E37F84A4BDD5B06B0970CC2D2BBB715F7B82846F9A0C393914C792E6A923E2117AB805276A975AADB5261D91673EA9AAFFEECBFA6183DFCB5D3B7332AA19275AFA1F8EC0B60FB6F66CC23AE4870791D5982AAD1AA9485FD8F4A60126FEB2CF05DB8A7F0F09B3397F3937F2E90B9E5B9C9B6EFEF642BC48351C46FB171B9BFA9EF17A961CE96C7E7A7CC3D3D03DFAD1078BA21DA425198F07D2481622BCE45969D9C4D6063D72AB7A0F08B2F49A7CC6AF335E08C4720E31476B6729E231F8BD90B39AC3AE3BE0C6B6CACEF8289A2E2873D58E51E029CAFBD55E6841489AB66B5B4B9BA6E2F784660896AFF387D92844CCB8B69475496DE19DA2

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E58259B0 90489AC8 E62363CD F82CFD8E F2A427AB CD65750B 506F56DD E3B98856 7A88126B 914D7828 E2B63A6D 7ED0747E C59E0E0A 23CE7D8A 74C1D2C2 A7AFB6A2 9799620F 00E11C33 787F7DED 3B30E1A2 2D09F1FB DA1ABBBF BF25CAE0 5A13F812 E34563F9 9410E73B

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